A New Approach to y-scaling and the Universal Features of Scaling Functions and Nucleon Momentum Distributions

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Abstract

Some systematic general features of y-scaling structure functions, which are essentially independent of detailed dynamics, are pointed out. Their physical interpretation in terms of general characteristics, such as a mean field description and nucleon-nucleon correlations, is given and their relationship to the momentum distributions illustrated. A new relativistic scaling variable is proposed which incorporates the momentum dependence of the excitation energy of the $(A-1)$ system, with the resulting scaling function being closely related to the longitudinal momentum distributions and being free from removal-energy scaling violating effects.

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Inclusive quasi-elastic electron scattering is potentially a powerful method for measuring the momentum distribution of nucleons inside a nucleus. Non-relativistically, this is most succinctly made manifest by expressing the data in terms of the scaling variable y which, over a large kinematic range, can be identified as the longitudinal momentum of the struck nucleon, $k_\parallel$ [1]. At sufficiently large momentum transfers, $\mathbf{q}$, the structure function, $W(\nu, q^2)$, which represents the deviation of the cross-section from scattering from free nucleons, scales to a function of the single variable y according to $qW(\nu, q^2) \approx f(y)$ where ν is the electron energy loss and $q \equiv |\mathbf{q}|$. Thus, in the scaling limit, qW approaches a function that effectively traces out the longitudinal momentum distribution of the nucleons:

$$f(y) = \int n(k_\parallel, \mathbf{k}_\perp) d^2\mathbf{k}_\perp = 2\pi \int_{|y|}^{\infty} n(k) k \, dk \tag{1}$$

Here, $n(k)$ (with $k \equiv |\mathbf{k}|$) is the conventional nucleon momentum distribution function normalized such that

$$\int d^3 k \, n(k) = \int_{-\infty}^{\infty} dy \, f(y) = 1 \tag{2}$$

Knowledge of $f(y)$ can therefore be used to obtain $n(k)$ by inverting Eq. (1):

$$n(k) = -\frac{1}{2\pi y}\frac{df(y)}{dy} \qquad\qquad |y| = k \tag{3}$$

The above picture relies on the simple assumptions that:

i) nucleon binding does not play a role in the scattering process; in reality, $qW(\nu, q^2)$ is determined by the spectral function, $P(k, E)$, which depends both on the removal energy (E), as well as on the momentum of the nucleons, through the relation [2] $qW(\nu, q^2) = F(y) = f(y) - B(y)$ where

$$B(y) = 2\pi \int_{E_{min}}^{\infty} dE \int_{|y|}^{k_{min}(y,E)} P_1(k, E) \tag{4}$$

P_1 being that part of $P(k, E)$ generated by ground state correlations (thus, in a mean field description or, for the case of ^2H, $P_1 = 0$) [3];

ii) final state interactions (FSI) are disregarded; if they are taken into account a direct relation between the experimentally measured $qW(\nu, q^2)$ and the asymptotic scaling function $F(y)$ holds only approximately.

Over the past several years there have been vigorous theoretical and experimental efforts to explore y-scaling over a wide range of nuclei [4], using a relativistic scaling variable resulting from energy conservation implicit in the instant form of relativistic dynamics (see Eq. 6 below) and representing the longitudinal momentum of those nucleons which have the minimal value of E (recently, it has been shown [5] that, for the deuteron, scaling in this variable $y(= y_r$ of ref. [5]) also follows from a relativistic light-front approach). In Ref. [2] the asymptotic scaling function $F(y)$ has been obtained by an elaborate extrapolation procedure of existing data aimed at removing (or, at least, minimising) the effects of FSI. The longitudinal momentum distribution $f(y)$ has thereby been obtained by adding to $F(y)$ the binding correction $B(y)$ evaluated theoretically. Such a procedure is based upon two basic assumptions: (i) the FSI can be represented as a power series in $1/q$, and die out at large q^2, a conclusion which has been reached by various authors [6] ;(ii) the theoretical binding correction has to be applied to obtain $f(y)$ [2]. Both assumptions, which in principle could be questioned, affect the final form of $f(y)$, and therefore need to be investigated. We will discuss the effects of FSI in a separate paper [7]; here we address only point (ii). To begin with, let us assume that $f(y)$ obtained in [2] is correct and let us analyze it in detail to see whether it contradicts or agrees with current theoretical predictions. A systematic analysis, to be presented elsewhere [7], exhibits the following general features of $f(y)$ for nuclei with $A < 56$:

i) $f(0)$ decreases monotically with A, from $\sim 10^{-2} MeV^{-1}$ when $A = 2$ to $\sim 3 \times 10^{-3} MeV^{-1}$ for ^{56}Fe; moreover, for $y \sim 0$, $f(y) \sim C_1(\alpha^2 + y^2)^{-1}$, with α (C_1) ranging from $\sim 45 MeV$ $(18 MeV)$ for $A = 2$, to $\sim 140 MeV$ $(59 MeV)$ for $A = 56$.

ii) For $50 MeV \leq |y| \leq 200 MeV$, $F(y) \sim e^{-a^2 y^2}$ with a ranging from $\sim 56 \times 10^{-2} MeV^{-1}$ for $A = 2$, to $\sim 45 \times 10^{-2} MeV^{-1}$ for $A = 56$.

iii) For $|y| \geq 400 MeV$, $f(y) \sim C_2 e^{-b|y|}$, with C_2 ranging from $2.5 \times 10^{-4} MeV^{-1}$ for $A = 2$, to $6 \times 10^{-4} MeV^{-1}$ for $A = 56$, and, most intriguingly, $b = 6 \times 10^{-3} MeV^{-1}$, *independent of A.*

The following form for $f(y)$ yields an excellent representation of these general features for all nuclei:

$$f(y) = \frac{C_1 e^{-a^2 y^2}}{\alpha^2 + y^2} + C_2 e^{-b|y|} \tag{5}$$

The first term ($\equiv f_0$) dominates the small y-behavior, whereas the second term ($\equiv f_1$) dominates large y. The systematics of the first term are determined by the small and intermediate momentum behaviours of the single particle wave function. For $|y| \leq \alpha$ this can be straightforwardly understood in terms of a zero range approximation and is, therefore, insensitive to details of the microscopic dynamics, or of a specific model. The small k behavior of the single particle wave function is controlled by its separation energy, ($Q \equiv M + M_{A-1} - M_A = E_{min}$) and is given by $(k^2 + \alpha^2)^{-1}$ so $\alpha = (2\mu Q)^{\frac{1}{2}}$, μ being the reduced mass of the nucleon.

Before discussing the intermediate range it is instructive to consider first the large y-behavior. Perhaps the most intriguing phenomenological characteristic of the data is that *f(y) falls off exponentially at large y with a similar slope parameter for all nuclei, including the deuteron.* Since (i) b is almost the same for all nuclei *including* $A = 2$, i.e., $f(y)$, at large y, appears to be simply the rescaled scaling function of the deuteron; and (ii) $b(\approx 1.18 fm) \ll 1/\alpha_D (\approx 4.35 fm)$, we conclude that the term $C_2 e^{-b|y|}$ is related to the short range part of the deuteron wave function and reflects the universal nature of NN correlations in nuclei. The remaining parameters, C_1 and a, can be related to $f(0)$ and the normalization condition, Eq. (2). Once this is done, there are no adjustable parameters for different nuclei. The intermediate range is clearly sensitive to a, with the gaussian form being dictated by the shell model harmonic oscillator potential, modulated, however, by the correct $|y| < \alpha$ behaviour, namely $(y^2 + \alpha^2)^{-1}$, thereby ensuring the correct asymptotic wave function. As an example, Fig. 1 shows the longitudinal scaling functions $f(y)$ for $^2 H$ and

^4He extracted from the experimental data ([2]), compared to Eq. (5). The errors of $f(y)$ for ^2H are very small, whereas, at large values of $|y|$ they are appreciable for ^4He and heavier nuclei; they are due both to the lack of reliable experimental data at large values of q^2, and to the necessity to correct the asymptotic scaling function for binding effects, and produce an error on C_2 and b in Eq. (5), ranging from $\simeq 6$ % to $\simeq 10$ %. A systematic analysis for a large body of nuclei exhibiting the same features as those shown in Fig. 1 will be presented elsewhere [7].

With these observations it is now possible to understand the normalization and evolution of $f(y)$ with A. First note that Eq. (1) implies $f(0) = \frac{1}{2} \int d^3k \frac{n(k)}{k} = \langle 1/2k \rangle$ and so is mainly sensitive to small momenta. Now, typical mean momenta vary from around 50 MeV for the deuteron up to almost 300 MeV for nuclear matter. We can, therefore, immediately see why $f(0)$ varies from around 10 for the deuteron to around 2-3 for heavy nuclei. More specifically, since $C_2 \ll C_1/\alpha^2$ and f_1 falls off so rapidly with y, the normalization integral, Eq. (2), is dominated by small y, i.e., by f_0. This leads to $f(0) \approx (\pi^{1/2}\alpha)^{-1} = (2\pi\mu Q)^{-1/2}$ which gives an excellent fit to the A-dependence of $f(0)$. Since $f(y)$ is constrained by the sum rule, Eq. (2), whose normalization is independent of the nucleus, a decrease in $f(0)$ as one changes the nucleus must be compensated for by a spreading of the curve for larger values of y. *Thus, an understanding of $f(y)$ for small y coupled with an approximately universal fall-off for large y, together with the constraint of the sum rule, leads to an almost model-independent understanding of the gross features of the data for all nuclei.*

To sum up, the "experimental" longitudinal momentum distribution can be thought of as the incoherent sum of a mean field shell-model contribution, (f_0), with the correct model-independent small y-behaviour built in, and a "universal" deuteron-like correlation contribution (f_1). Thus, the momentum distribution, $n(k)$, which is obtained from (3), is also a sum of two contributions: $n = n_0 + n_1$. This allows a comparison with results from many body calculations in which n_0 and n_1 have been separately calculated. Of particular relevance are not only the shapes of n_0 and n_1, but also their normalizations, $S_{0(1)} \equiv \int n_{0(1)} d^3k = \int f_{0(1)} dy$ which, theoretically, turn out to be, for ^4He, $S_0 \sim 0.8$ and $S_1 \sim 0.2$ [8]

whereas Eq. (5) yields $S_0 = 0.76$ and $S_1 = 0.24$. A comparison between the momentum distributions obtained from y-scaling and the theoretical ones is shown in Fig. 2. As can be seen the $n(k)$ compare very well with theoretical calculations. Our analysis confirms very well known properties of low energy nuclear physics (binding, etc), as well as some features of the momentum distributions predicted recently by various theoretical calculations [8], [9], [10], but still waiting for a firm experimental confirmation. To place our results on a more solid basis, it would be necessary however to reduce the errors on $f(y)$ at large values of $|y|$, which necessitates experimental data at larger values of q^2, as well as a reduction of the uncertainties related to the binding correction. Experimental data at higher Q^2 became recently available [19], and their inclusion in the analysis reduces indeed the errors on $f(y)$ [7]. Here we address the problem of making the extraction of $f(y)$ as much independent as possible from theoretical binding corrections. This is accomplished by introducing another scaling variable. A generic ambiguity in defining a scaling variable is that there is no unique prescription for y, so that it is legitimate, in principle, to incorporate in the definition of y some physical dynamical effects by introducing proper effective parameters. The usual scaling variable y is obtained from relativistic energy conservation

$$\nu + M_A = [(M_{A-1} + E^*_{A-1})^2 + \mathbf{k}^2]^{1/2} + [M^2 + (\mathbf{k} + \mathbf{q})^2]^{1/2} \qquad (6)$$

by setting $k = y$, $\frac{\mathbf{k} \cdot \mathbf{q}}{kq} = 1$, and, most importantly, the excitation energy, $E^*_{A-1} = 0$; thus, y represents the nucleon longitudinal momentum of a nucleon having the *minimum value of the removal energy* $(E = E_{min}, E^*_{A-1} = 0)$. The minimum value of the nucleon momentum when $q \to \infty$, becomes $k_{min}(y, E) = |y - (E - E_{min})|$. Only when $E = E_{min}$ does $k_{min}(y, E) = |y|$, in which case the binding correction $B = 0$ and $F(y) = f(y)$. However, the final spectator $(A - 1)$ system can be left in all possible excited states, including the continuum, so, in general, $E^*_{A-1} \neq 0$ and $E > E_{min}$, so $B(y) \neq 0$, and $F(y) \neq f(y)$. Thus, it is the dependence of k_{min} on E^*_{A-1} that gives rise to the binding effect, i.e. to the relation $F(y) \neq f(y)$. This is an unavoidable defect of the usual approach to scaling; as a matter of fact, the longitudinal momentum is very different for weakly bound, shell model nucleons (for which $E^*_{A-1} \sim$

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$0 - 20 MeV$) and strongly bound, correlated nucleons (for which $E_{A-1}^* \sim 50 - 200 MeV$), so that at large values of $|y|$ the scaling function is not related to the longitunal momentum of those nucleons (the strongly bound, correlated ones) whose contributions almost entirely exhaust the behaviour of the scaling function. In order to establish a global link between experimental data and longitudinal momentum components, one has to conceive a scaling variable which could equally well represent longitudinal momenta of both weakly bound and strongly bound nucleons. An attempt in such a direction has been made in the past by [14] and, recently, by us [15] with some minor differences with respect to Ref. [14] attempt. This was based upon taking literally the two-nucleon correlation model according to which the large k and E behaviours of the Spectral Function are governed by configurations in which the high momentum of a correlated nucleon (1, say) is almost entirely balanced by another nucleon (2, say). Within such a picture, one obiously has $E_{A-1}^* = \frac{A-2}{A-1}\frac{1}{2M}\mathbf{k}^2$ and the average excitation energy for a given value of k is $< E_{A-1}^*(k) >= \frac{A-2}{A-1}\frac{\mathbf{k}^2}{2M}$. By replacing E_{A-1}^* in Eq. (6) with $< E_{A-1}^*(k) >$, the deuteron-like scaling variable y_2 introduced in our previous paper [15](see also [14], where a similar scaling variable was first introduced) is obtained, representing the scaling variable pertaining to a "deuteron" with mass $\tilde{M} = 2M - E_{th}^{(2)}$, where $E_{th}^{(2)} = M_{A-2} + 2M - M_A$. Such a scaling variable, however, has the unpleasant feature that the effect of the deuteron- like correlations are overestimated at low values of y_2 and , as a result, the correct shell-model picture provided by the usual variable y is lost.

In a more refined model, the CM motion of the pair is taken into account: one has [11]

$$E_{A-1}^* = \frac{A-2}{A-1}\frac{1}{2M}[\mathbf{k} - \frac{A-1}{A-2}\mathbf{K}_{CM}]^2 \qquad (7)$$

which shows that the excitation energy of the residual nucleus depends both upon $\mathbf{k}$ and $\mathbf{K}_{CM}$; although using Eq. (7) the situation is improved, the description ot low values of y is not a satisfactory one. In this paper we *assume* that the nucleus is described by a realistic spectral function as provided by few- and many-body calculations [12], [13] and implement such a realistic description into the definition of the scaling function. If the expectation value of Eq. (7) is evaluated with realistic spectral functions, for the three-body system [12]

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and nuclear matter [13] and the model spectral function of [11] one obtaines ([7,11])

$$< E^*_{A-1}(k) >= \frac{A-2}{A-1}\frac{1}{2M}\mathbf{k}^2 + b_A - c_A\frac{1}{2M}\mathbf{k}^2. \tag{8}$$

Here, b_A and c_A, resulting from the CM motion of the pair, have values ranging from $17 MeV$ to $43 MeV$ and 3.41×10^{-1} to 1.66×10^{-1}, for ^3He and Nuclear Matter, respectively. Placing Eq.(8) in Eq.(6) and subtracting the value of the average removal energy $< E >$ to counterbalance the value (8) at low values of y, a new scaling variable is obtained. This effectively takes into account the k-dependence of the excitation energy of the residual $(A-1)$ system, both at low and high values of y. This is in contrast to the usual scaling variable, which completely disregards E^*_{A-1}, and the scaling variable y_2, which overestimate the effects of deuteron-like correlations at low values of y. In the kinematical region of existing experimental data this new , *global* scaling variable, y_{CW}, that we have obtained, reads as follows

$$y_{CW} = \left| -\frac{\tilde{q}}{2} + \left[\frac{\tilde{q}^2}{4} - \frac{4\nu_A{}^2 M^2 - W_A{}^4}{4W_A{}^2} \right]^{1/2} \right| \tag{9}$$

Here, $\nu_A = \nu + \tilde{M}$, $\tilde{M} = (2A-3)M/(A-1) - E^{(2)}_{th} - (b_A + 2M^2 c_A - < E >)$, $\tilde{q} = q - c_A\nu_A$ and $W_A^2 = \nu_A{}^2 - \vec{q}^2 = \tilde{M}^2 + 2\nu\tilde{M} - Q^2$. For the deuteron $E^*_{A-1} = 0$, so $y_{CW} \rightarrow y = |-q/2 + [q^2/4 - (4\nu_d{}^2 M^2 - W_d{}^4)/W_d{}^2]^{1/2}|$ with $\nu_d = \nu + M_d$ and $W_d^2 = \nu_d{}^2 - \vec{q}^2 = M_d{}^2 + 2\nu M_d - Q^2$. For small values of y_{CW}, such that $(\frac{A-2}{A-1}\frac{1}{2M}y^2 + b_A - c_A\frac{y^2}{2M}) \lll < E >$, the usual variable, representing the longitudinal momentum of a weakly bound nucleon is recovered [21]. Thus y_{CW} interpolates between the correlation and the single particle regions. More importantly, however, since $k_{min}(q,\nu,E) \simeq |y_{CW}|$, $B(y_{CW}) \simeq 0$, $F(y_{CW}) \simeq f(y_{CW})$. One would therefore expect from our above analysis, the same behaviour of $f(y_{CW})$ at high values of y_{CW} for both the deuteron and complex nuclei (unlike what happens with the usual scaling function $F(y)$), and the same shell-model behaviour at low values of y, as predicted by the usual scaling variable. This is, indeed, the case, as exhibited in Figs. 3 and 4, where the direct, global, and independent of A link between the scaling function $F(q, y_{CW})$ and the longitudinal momentum distributions is manifest .

We can summarise our conclusions as follows:

i) The general universal features of the y-scaling function have been identified and interpreted in terms of a model-independent zero-range contribution, a "universal" 2-nucleon correlation contribution and a mean field (shell-model) contribution. The shape and evolution of the curve have been understood both quantitatively and qualitatively on general grounds.

ii) A global relativistic scaling variable which, unlike all previously proposed variables, incorporates the average mean field excitation energy of the $(A-1)$ system, as well as the excitation energy produced by deuteron-like correlated pairs and by their center-of-mass motion in the nucleus, has been defined. Such a variable, thanks to these new features, allows one to establish a more direct link between the scaling function and the longitudinal momentum distributions at any value of y. Thus, using this variable, it would be possible in principle to obtain the longitudinal momentum distributions directly without introducing theoretical binding corrections. Of course the usual variable has the advantage of being defined in terms of a well-defined experimental quantity, the minimum value of the removal energy E_{min}, whereas y_{CW} incorporates the excitation energy of $(A-1)$ by a theoretical prediction. However, given the fact that in $\gamma^* - Nucleus$ scattering the virtual photon couples to nucleons having different values of the removal energy, suggests that the removal energy has to be taken into account in the definition of the scaling variable. Our Fig. 3 shows indeed that removal energy effects are very important. Since these are a source of scaling violation, the other source being the FSI, it seems reasonable to incorporate the binding effects into the definition of y so as to ascribe the remaining scaling violation to FSI. Moreover the plot of the data in terms of y_{CW} has a very clear cut meaning: $F(y_{CW}, q^2)$ represents the scaling function at a given q^2 and for a longitudinal momentum of a nucleon having removal energy $E_{(y_{CW})} = E_{th}^{(2)} + \frac{A-2}{A-1} \frac{1}{2M} y_{CW}^2 + b_A - c_A \frac{1}{2M} y_{CW}^2$. As already pointed out, there is no unique prescription for defining a y-scaling variable, and various variables

are currently being used [17]. Ultimately, a valid criterion for a scaling variable, is to produce scaling; the usual y has been shown to produce scaling but, at large values of $|y|$, in the ultra asymptotic limit [2], [18], [5], that is a limit which is not reached by present experimental data. In Ref. [7], it is shown that the new scaling variable not only produces precocious scaling to the longitudinal momentum distribution even at the largest value of $|y|$ recently reached by the new TJLAB experimental data [19], but also that the FSI on the scaling function $F(y_{CW}, q^2)$ are very similar to the ones acting in the deuteron. Therefore, in terms of this variable the data seem to support the idea that the large y_{CW} behaviour in all nuclei is essentially nothing but a rescaled version of the deuteron (including, perhaps, also some effects from deuteron-like FSI, due to the constant value of the nucleon-nucleon cross section in the region $2 \leq Q^2 \leq 6$ GeV^2, as stressed in [20]). Such a conclusion cannot be reached by analysing the data in terms of the old scaling variable, which mixes up scaling violation due to removal energy effects and FSI.

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[21] Note that Eq. (9) does not contain free parameters, since b_A and c_A are determined from the Spectral Function and $< E >$ from the Koltun sum rule [11]

FIGURES

FIG. 1.   The "experimental" longitudinal momentum distributions of ^2H and ^4He obtained [7] using the results of [2] compared with Eq. (5) with $\alpha = 45MeV$, $C_1 = 18MeV$, $a = 56 \times 10^{-2} MeV^{-1}$, $C_2 = 2.5 \times 10^{-4} MeV^{-1}$, and $b = 6 \times 10^{-3} MeV^{-1}$, for ^2H, and $\alpha = 167MeV$, $C_1 = 106MeV$, $a = 68.5 \times 10^{-2} MeV^{-1}$, $C_2 = (6 \pm 0.6) \times 10^{-4} MeV^{-1}$, and $b = (6 \pm 0.6) \times 10^{-3} MeV^{-1}$, for ^4He

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FIG. 2. The nucleon momentum distribution for ^2H from Eq.(6), compared with the one obtained from the AV14 interaction; the same as Fig. 2a but for ^4He

FIG. 3.   The experimental scaling function $F(q, y)$ for ^2H, ^4H and ^{56}Fe compared with the longitudinal momentum distributions $f(y)$ given by Eq.(5) (dot-dash-^2H; short-dash-^4H; full-^{56}Fe). The scaling variable is the usual one, i.e. the one obtained from Eq.(5) placing $E^*_{A-1} = 0$

FIG. 4.   The same as in Fig.3, but with the variable , y_{CW}, obtained using Eq.(8)
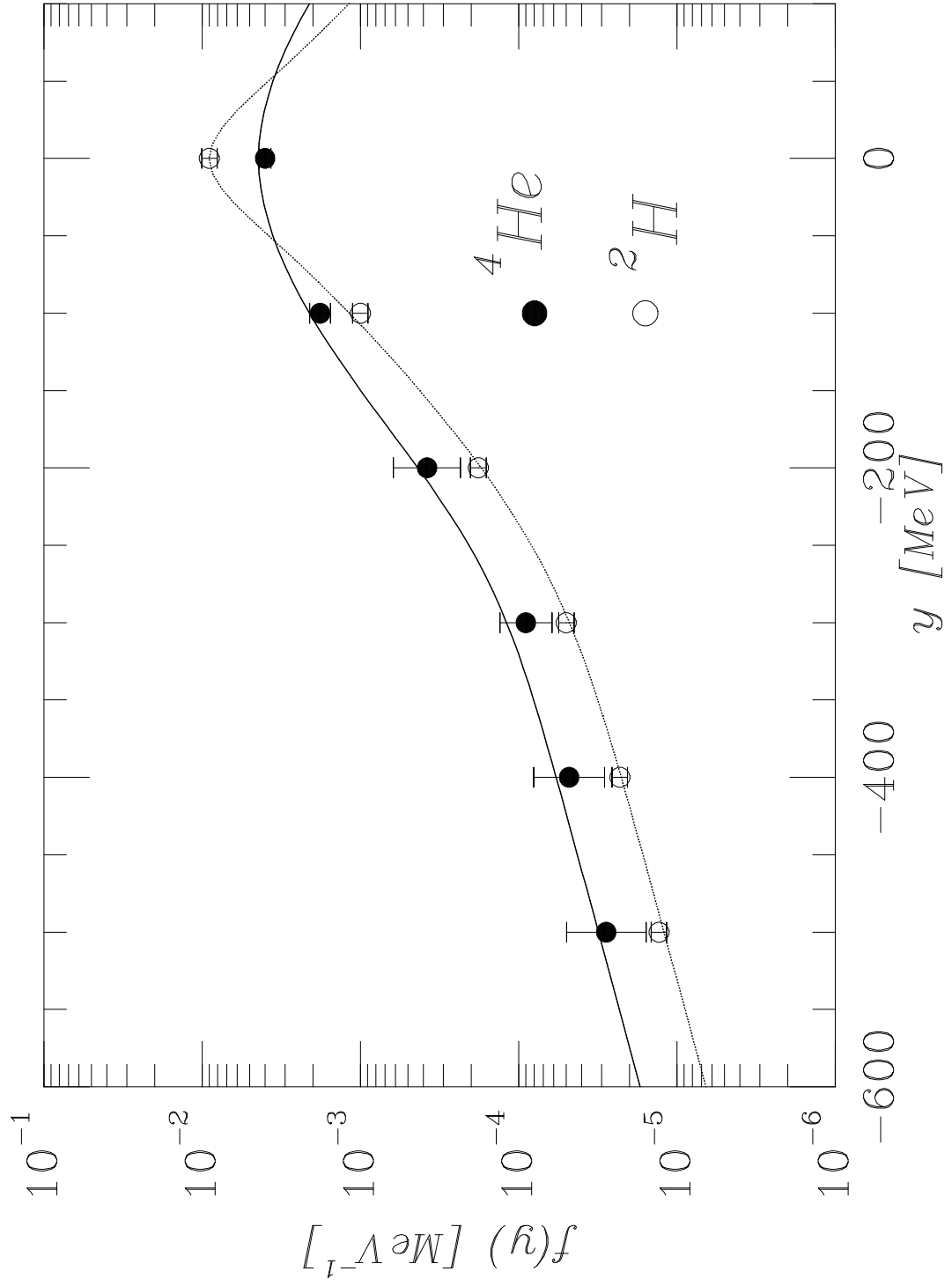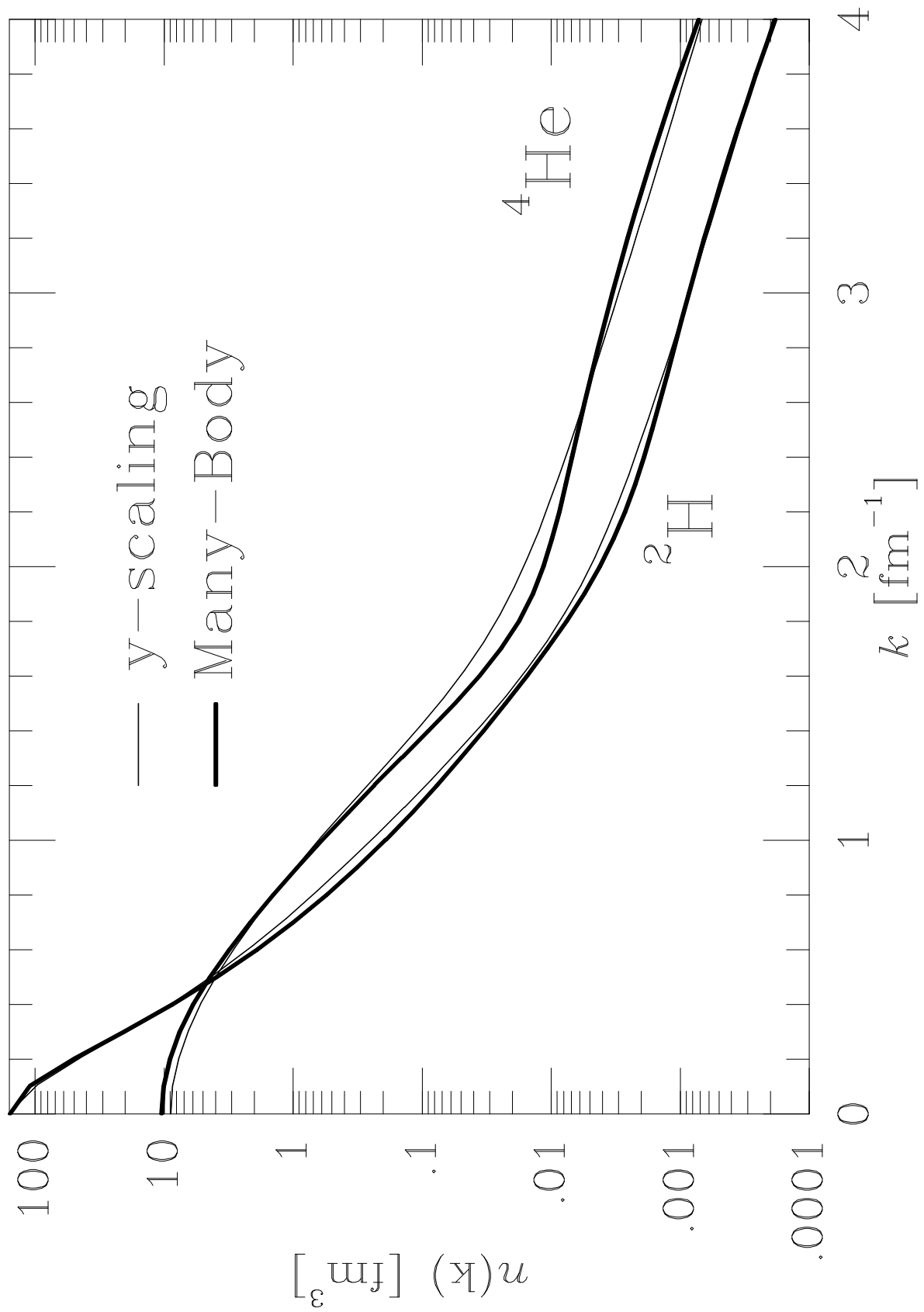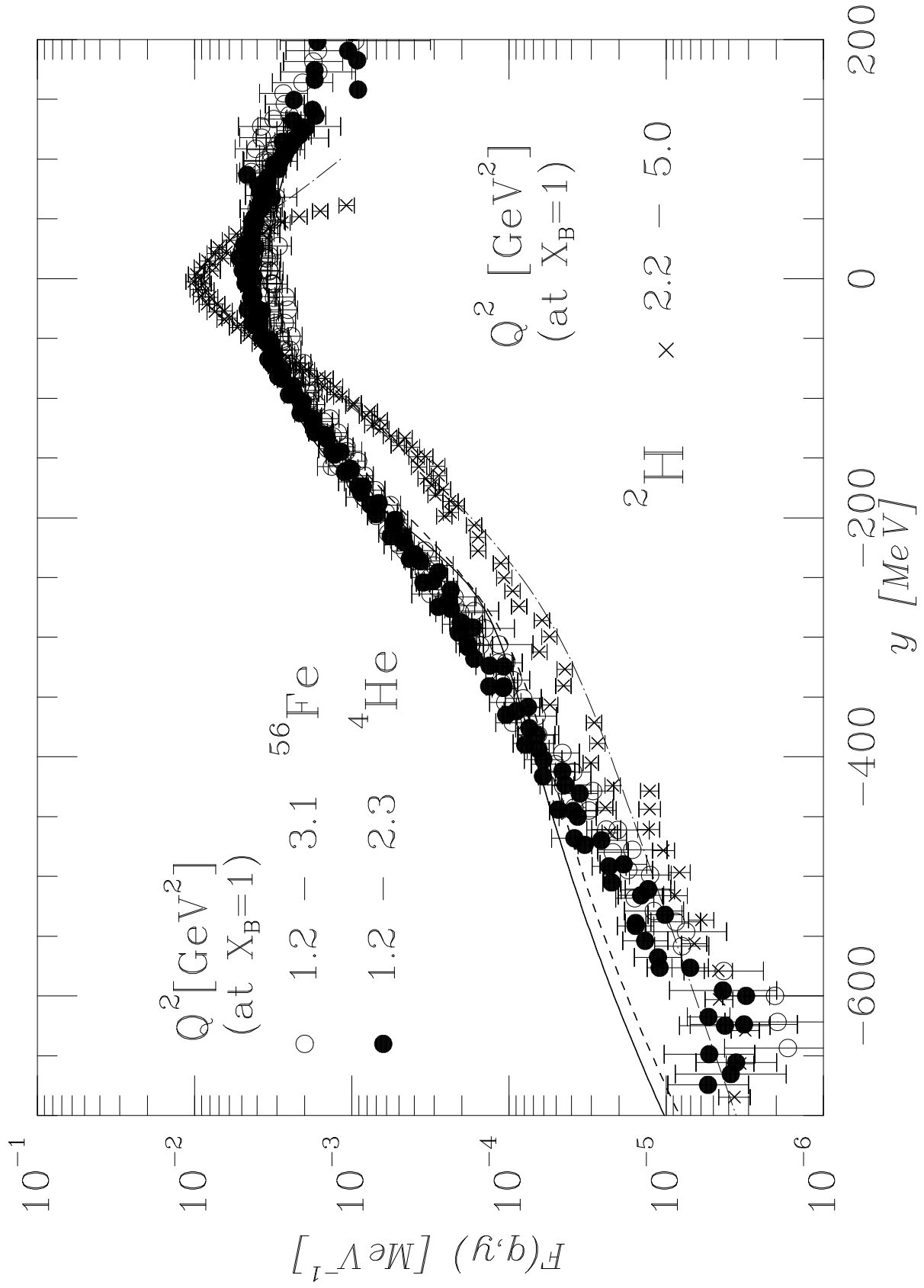
Fig. 1. Ciofi-West Y-scaling

Fig. 2. Ciofi-West Y-scaling

Fig. 3. Ciofi-West Y-scaling

$Q^2 [GeV^2]$
(at $X_B=1$)

^{56}Fe

^4He

$\bigcirc$ $1.2 - 3.1$

$\bullet$ $1.2 - 2.3$

$Q^2 [GeV^2]$
(at $X_B=1$)

^2H $\times$ $2.2 - 5.0$

$y \; [MeV]$

$F(q,y) \; [MeV^{-1}]$

10^{-1}
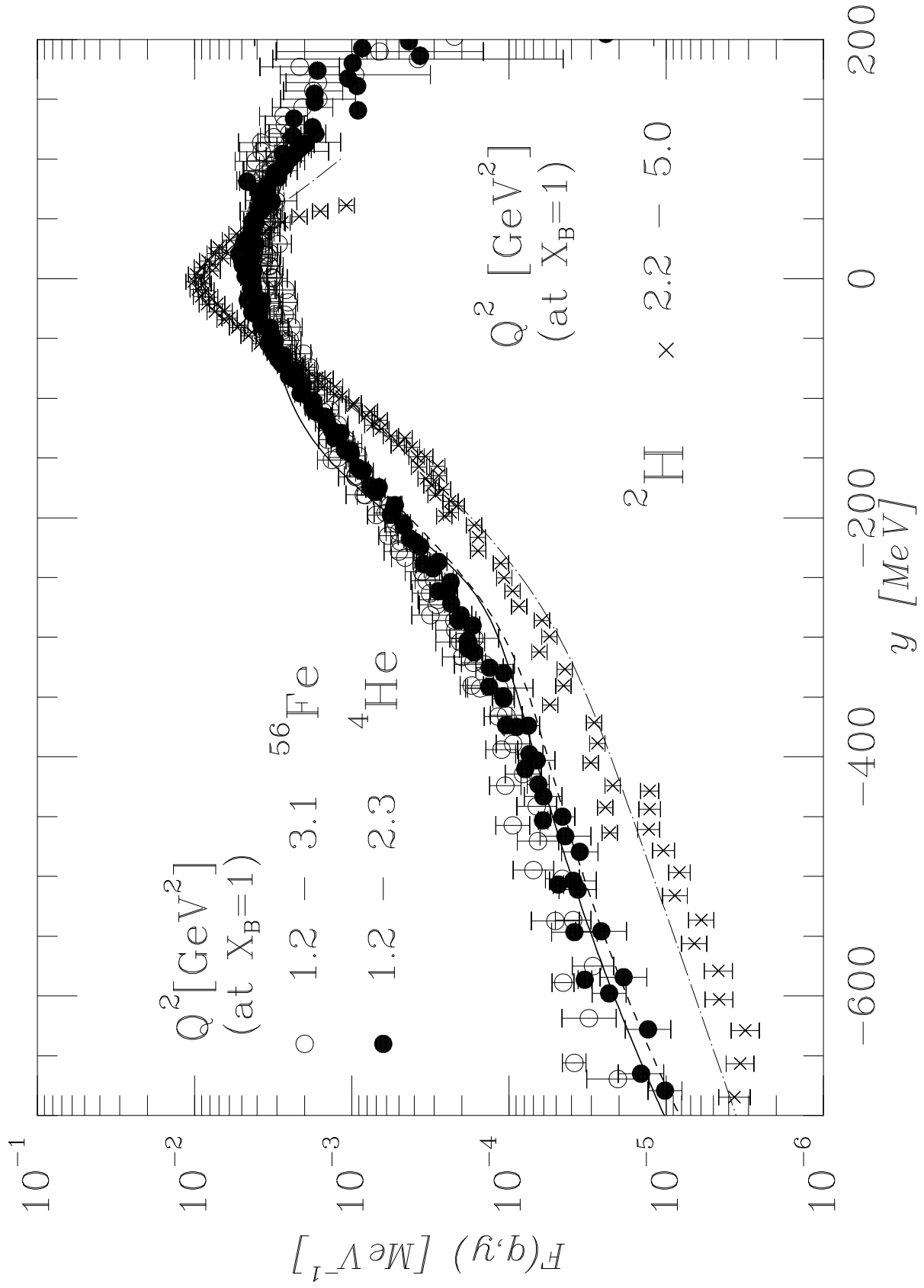
10^{-2}

10^{-3}

10^{-4}

10^{-5}

10^{-6}

200

0

-200

-400

-600

Fig. 4. Ciofi-West Y-scaling